Team Control Number

10876

Problem Chosen

B

2020
HiMCM
Summary Sheet

Limited Budget, Unlimited Future

With rapid environment deterioration and biodiversity shrinkage, conservation of species has become a global issue in the 21st century. However, since a large number of species require conservation and the capacity of fundraising is limited for institutions like FRPCE, it is impossible to start all projects simultaneously, and some must be postponed. Postponing projects cause a decrease in feasibility of success. In this paper, by optimizing the priority order of the projects and a producing a timetable of projects, we will advise the FRPCE Board of the minimum fundraising required each year.

First, we analyzed some common characteristics of imperiled plants, of which we picked 5 that are closely related to our decision model: benefit, taxonomic uniqueness, feasibility of success of conservation, timespan, and total cost of a project. We also identified and discussed the objectives that the FRPCE Board should consider in their conservation efforts and budgeting decisions, including lowering the cost, saving more plants, and finishing the projects sooner.

Second, we defined the concept of "cost effectiveness." For an individual project, the cost effectiveness is determined by the benefit, the feasibility of success and the total funding required. The term is defined by the expected benefit brought by unit money. Cost effectiveness balances and quantifies the somewhat contradictory objectives of FRPCE. The sum of the individual cost effectiveness is the integrated cost effectiveness, a criterion of the efficiency of capital allocation, which should be maximized.

Third, we built two models. One was the Feasibility of Success Model, which was used to model how feasibility of success varies with time, and the other was the Optimization Model for Integrated Cost Effectiveness, whose constraint was the budget limit. To solve the optimization model, we designed an algorithm based on Monte Carlo Method and presented a demonstration.

Forth, our model outputted a timetable, which showed which projects were ongoing in what year and how much they cost. Thus, we could obtain the priority order of the projects and the fundraising schedule. When the budget limit is constant at $1.05M each year, we obtained a solution that has a timeline of 31 years. The projects with ID 528, 179, 455, 524, 543, 133, 513, 546, 442, 480, 536 and 532 started in the first year, and the budget required in the first year was $1034015.58. Unfortunately, we were forced to give up "Flowering Plant-551" because the conservation cost far exceeded the annual budget limit. If the annual budget limit is linearly increasing each year rather than constant, the conservation of "Flowering Plant-415" would be feasible. We also obtained an optimized solution for this case.

Last, we carried out sensitivity analysis on the annual budget, the rate at which annual budget is increasing, and the rate at which the populations of imperiled plants shrank. Our model reasonably reflected the influence of the variables while maintaining stability and applicability. According to the result of the sensitivity analysis for the annual budget, we further concluded that if FRPCE could raise 1.3 times the money it is currently raising annually, the cost effectiveness would increase substantially, and we provided a timetable of projects in this case as well.

Keywords: Imperiled Plants, Cost Effectiveness, Objective Optimization, Monte Carlo Method

To: the FRPCE Board

From: Team #10876

Date: Nov 15th, 2020

Subject: Fundraising shedule, mathematical models

#10876

Limited Budget, Unlimited Future

Honorable FRPCE Board,

It is our pleasure to share our advises regarding the fundraising schedule and the priority order of conservation projects. The work of FRPCE is globally beneficial and truly outstanding. As human society develops and expands, the environment deteriorates, rendering numerous plants imperiled or extinct. However, the budget is limited for us. But fortunately, we already have found a solution.

To reasonably control the fundraising difficulty for FRPCE, we set an upper limit of annual budget. The fund required each year cannot be over this limit. Apparently, the 48 programs cannot start simultaneously under the limit, so some must be postponed for 1 year, 2 years, or more. The population of the postponed species will keep shrinking, and the feasibility of success of conservation will diminish, which is undesired.

In short, the objective of FRPCE is to save species, as many as possible and as soon as possible, under limited budget. These goals have to be balanced. We defined the "Cost Effectiveness" to incorporate these three variables. For a conservation program, the Cost Effectiveness is the expected benefit per unit cost. We aimed at maximizing it under the budget limit. Thus, the limited capital is fully utilized to preserve species.

We built two models: One is Feasibility of Success Model, which is used to model how feasibility of success varies with time, and the other is the Optimization Model of Integrated Cost Effectiveness, whose objective function is the integrated cost effectiveness, and the restraint is the budget limit. According to our models, we obtained the optimized timetable of programs and also the fundraising schedule and the priority order of conservation programs.

Finally, we proposed two solutions as follow:

1. **First solution:** we consider budget limit as a constant, which does not vary among years. When the annual budget limit is $1.05M which determined according to the 2019 data on the official website of FRPCE, our model suggests starting programs with ID 528, 179, 455, 524, 543, 133, 513, 546, 442, 480, 536 and 502 in the first year. The funding required in the first year is $1034015.58. All project will conclude in 31 years. Unfortunately, we gave up "Flowering Plant-415" because its annual cost exceeds our budget limit. If FRPCE is able to raise more money annually, more programs can start earlier, facilitating the feasibility of success. If budget limit could raise 1.3 times as much ($1.71M per year), the integrated cost effectiveness would increase substantially, and it is strongly recommended. The optimized timetable of programs and other details are shown in Section 6 and Section 7.
2. **Second solution:** The two cases above both regarded the budget limit as a constant. However, if the FRPCE Board is confident that more funding will be available each year, we also produced a solution under a linearly increasing budget. An advantage of an increasing budget limit is that we are able to protect "Flowering Plant-415" after the annual budget increases to a certain amount. For more details, please see Section 6.3.2.

Content

1. Introduction

1.1. Background

Whereas human society is developing at an unprecedented pace, plants and animals is left to face the disastrous environmental consequences. Legislations, as well as fund donations and public campaigns, help fighting plant and animal extinctions. Signed into law in 1973 by President Richard Nixon, the Endangered Species Act (ESA) is credited with saving America's national animal — as well as the California condor, grizzly bear, northern gray wolf, and more. Today it protects more than 1,600 plant and animal species and 99% of the species placed on the endangered list have not gone extinct, says Jeremy Bruskotter, a professor at the School of Environment and Natural Resources at Ohio State University. [1] Also, because existing plant conservation funding is inadequate to support research, protection, and management of imperiled plants, a group of conservation specialists representing seven institutions in Florida began conversations in 2015 to initiate the Florida Rare Plant Conservation Endowment (FRPCE). The FRPCE is being established as a mechanism to provide long-term and reliable funding to support conservation-related projects for Florida imperiled plant species and their ecosystems. [2] Even so, the fund raised annually is far below the capital required to run all the protection projects of endangered plants.

1.2. Question Restatement

The general purpose of our work was to construct a model to give FRPCE Board an effective and reliable fundraising plan. Due to considerations of complexity of this decision-making problem, the scope of analysis was limited to only 48 imperiled species.

- First, we should focus on the relevant objectives in our fundraising model. In the following work, we then should try to find a method to evaluate whether a proposed fundraising plan is efficient and reliable on the basis of these objectives.
- Second, we should list and explain some general features of the imperiled species. We then need to determine the factors involved in the imperiled species' protection in fundraising decision model.
- Third, we need to develop a model that optimizes the arrangement of projects and output the solution as a timetable and a fundraising schedule.
- Last, we need to write a one-page non-technical memo, explain our results, and then give our recommendations on our model and analysis.

1.3. Our Work

We analyzed the characteristics of imperiled plants and the objectives of FRPCE, and incorporated and quantified them into the "integrated cost effectiveness." Then we built an optimization model, which also took the variation of feasibility of success with time (Model of Feasibility) into consideration. Afterwards, the model was solved by Monte Carlo Method. Finally, the model outputted the schedule and priority order of projects, and a fundraising schedule as well. The process is shown in the flowchart in Fig. 1.

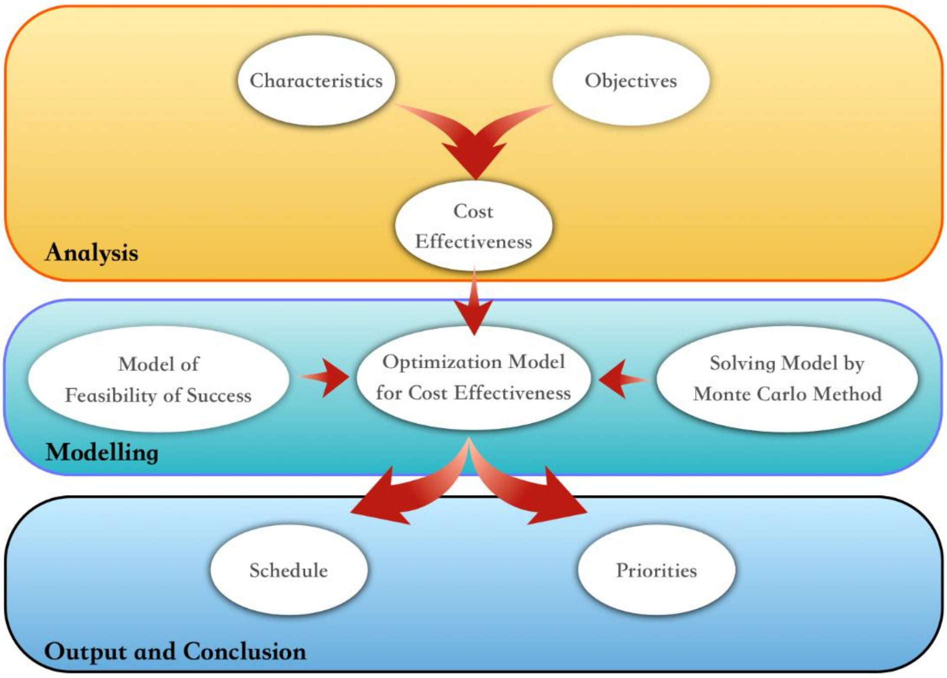

Fig. 1 Framework of This Paper

We also varied the inputs of the model and produced results under different circumstances: a higher budget limit or a linearly increasing budget limit. We provided further suggestions based on the results.

2. Assumptions

2.1. Assumptions and Justifications

- **Assumption 1:** FRPCE has a maximum capacity of fund raising each year. Also, FRPCE raises as much as the conservation projects that year cost, i.e., it cannot save money.
  Justification 1: FRPCE cannot make money by itself, and it relies on government and public donations. Its campaigns are unable to Donors and government officers would demand a financial report from FRPCE that demonstrates the use of the capital. The fund-raising must be completely need-based, and no excess capital is available to be passed on to the next year.

- **Assumption 2:** The maximum capacity of fund raising of FRPCE does not allow all the projects to start immediately. Some must be postponed, until more funding arrives in the future.
  Justification 2: If all the projects start simultaneously, the cost is more than ten times the money FRPCE put into plant conservation in the past year. [3] It is, apparently, unrealistic.

- **Assumption 3:** When a conservation project is postponed, the total cost and the benefit of successful conservation do not change, but the feasibility of success decreases. The rate of decrease is the same among all species.
  Justification 3: The total cost does not change because the approach of the project remains roughly the same. Also, in a relatively short time span (about 30 years,) the extent of uniqueness and benefit of the existence of a species is almost unchanging. The feasibility of success decreases because the size of population of the species decreases constantly until it becomes extinct. A smaller population means less offspring and less generic variation to counter environmental challenges.

- **Assumption 4:** A project has to be funded continuously from the beginning to the end.
  Justification 4: If the project abruptly stops, the plants previously protected would be facing the threats again. The feasibility of success drops massively. For some conservation projects that involve domesticating plants, those plants have to be continuously looked after, or the consequence would probably be disastrous for them and for the species.

- **Assumption 5:** Species are independent, i.e., the survival of one species does not affect the situations of the others.
  Justification 5: The imperiled plants are relatively small in number. Their populations are not large enough to significantly influence each other.

2.2. Definitions

- **Fundraising Schedule**
- The fundraising schedule refers to the amount of funding required each year.

- **Timetable of Projects & Priority Order of Projects**
- The timetable of projects refers to the arrangement that shows which project starts or is ongoing in which year; the priority order of projects shows a linear order in which the projects are carried out, but it does not indicate when each project starts.

- **Arrangement of Projects**
- See Timetable of Projects.

- **Budget & Budget Limit**
- The budget is the funding required by the ongoing conservation projects that year (see Assumption 1;) the budget limit refers to the maximum feasible budget per year, determined by the capacity of fundraising of FRPCE.

2.3. Notations

Table 1 Notations

Notation	Definition
j	The j^{th} year
B_i	Benefit of the i^{th} project
T_i	Timespan of the i^{th} project
k_i	The i^{th} project starts in the k_i^{th} year
$S_i(k_i)$	The feasibility of success of the i^{th} project which starts in the k_i^{th} year
P_i	Total cost of the i^{th} project
$f_i(n)$	Funding required by the i^{th} project in its n^{th} year, $n \in N^*$
u_j	Maximum capacity of fund raising of FRPCE in the j^{th} year
N	Total number of projects

3. Question Analysis

We analyzed the characteristics of the plants and the objectives of FRPCE and aimed at an optimization model incorporating them.

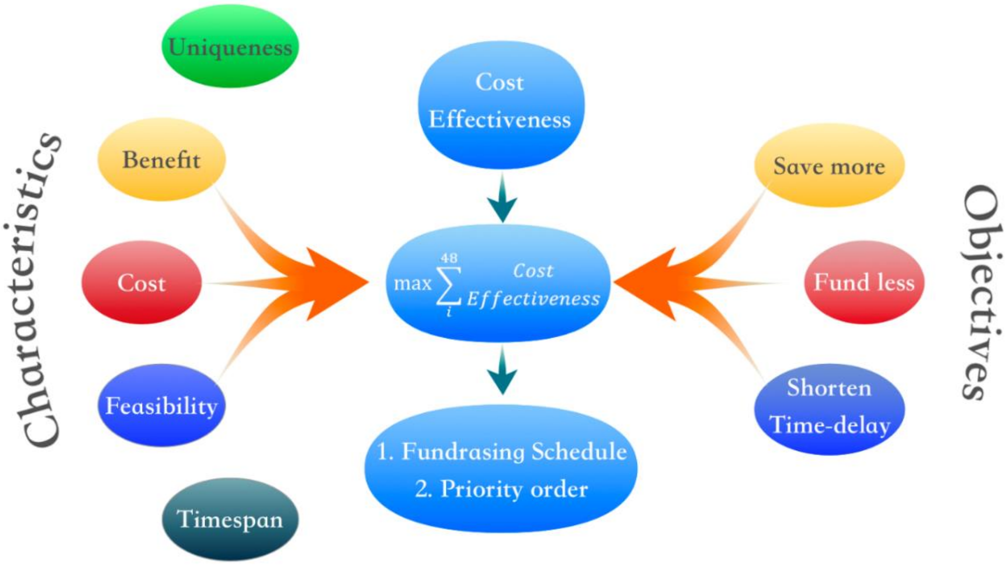

Fig. 2 Incorporating Characteristics of Plants and Objectives of FRPCE

3.1. Characteristics of Imperiled Plants

(For question 1b)

There are several characteristics that are indicative when deciding which species to recover:

- **Benefit of saving the plant:** This index implies relatively how beneficial saving this plant is. Plants with higher benefit are prioritized in conservation.
- **Taxonomic uniqueness of the plant:** The more unique a plant is, the more it contributes to biodiversity. Plants with high taxonomic uniqueness should be prioritized.
- **Feasibility of Success of Conservation:** Imperiled plants are often in harsh situations. Their conservation and recovery are often challenging, and success cannot be guaranteed. If a project fails after being invested, the funding is all wasted. Projects with high feasibility of success are prioritized.
- **The timespan of protecting a species:** Longer timespan leads to higher uncertainties of capital chain break, which cause the project to fail, and usually more funding is required by these projects. Conservation projects with shorter timespan are favored.
- **The total cost of a protection project:** Imperiled plants are delicate, and the protection of them is intricate, sophisticated and therefore, expensive. In contrast, the institutions that carry out the conservation projects often relies heavily on donations, and their capital is limited. Therefore, the less costly projects should be prioritized.

3.2. Objectives of FRPCE

(For question 1a)

We believe that the relevant objectives FRPCE Board need to consider in species protection and budgeting decision are the following.

- To save more species.

- To lower the amount of annual fund raised to lighten the fundraising burden.
- To shorten the time delay for conservation projects as much as possible: since the longer the time delay for a given project, the smaller the size of population and the lower the chance for that targeted species to survive.

However, these objectives above are mutually contradictory. For instance, limiting the budget each year indicates that there are fewer available projects, less benefits obtained, more project being postponed, and lower the chance of success of protecting species from extinction.

Thereout, we define a new index: integrated cost effectiveness, which evaluates the expected benefits obtained per unit capital per project and integrates benefits, feasibility of success and the cost for a single project together. Aiming at integrated cost effectiveness, we can plan a better fundraising schedule. This index will be elaborated into more details in section 4.2.

3.3. Fundraising Schedule and Priority Order

In order to determine the priority order of funding for the recovery projects and the corresponding fundraising schedule, the data given need to be closely examined. We especially focused on the Uniqueness, benefit and the annual cost of each recovery project.

- The upper-left part of Fig. 3 shows that there are only 3 types of Uniqueness: 1, 0.67 and 0.33. The major plants have uniqueness of 0.67. Only 2 are the most unique and 3 are least unique. We could first recover the most unique ones and then others later when solving the model.
- The benefit of projects consists of 4 different values. More than half of the projects have benefit of 0.66. We protect the plants with higher benefit first because we want to have maximized benefits, and postponing will lead to decrease of overall benefit.
- Also, "Flowering Plant-415" costs substantially more than the others as represented as the highest yellow line in right part of Fig. 3. Other than the outlier, the figure shows a trend of decline in cost for each project as time passes. So, we can imply that the cost needed will decrease once some projects started. Thus, putting off some projects to later years and wait until there is enough fund should be a workable method. However, the feasibility of a project will decrease if it is put off. In other words, the cost effectiveness gets lower when it starts late—this is not desired.

We will set up a model that maximizes the sum of the cost effectiveness of the conservation project of 48 plants. The model will be solved under the constraint of annual budget. The budget for every year and the recovery projects that are needed to be started in each year will be presented in the solution.

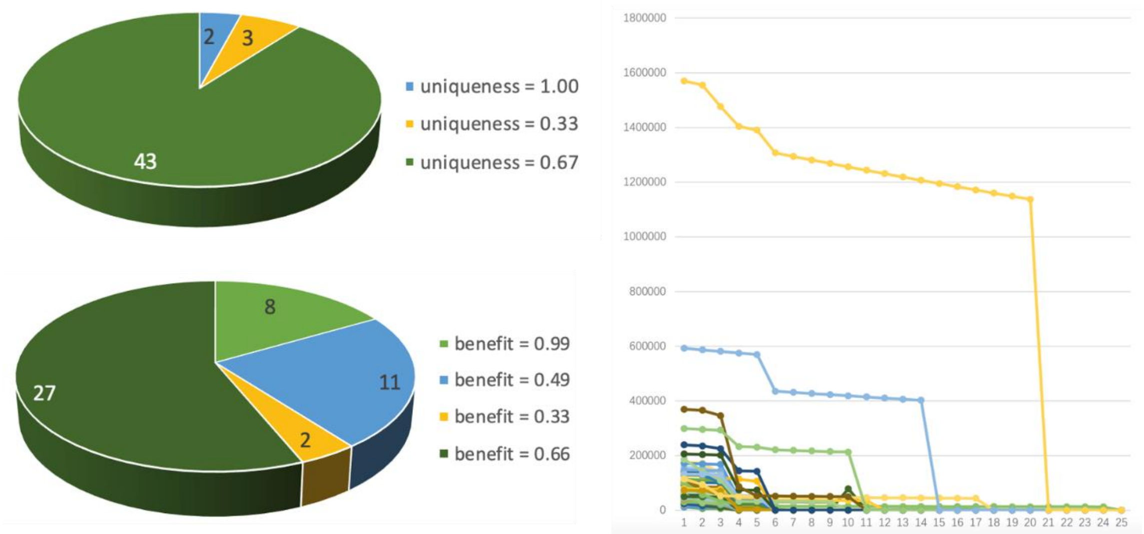

Fig. 3 Statistics of Taxonomic Uniqueness, Benefit and Cost

4. Optimization Model for Integrated Cost Effectiveness

The model aims at incorporating the characteristics of the imperiled plants and the objectives of FRPCE mentioned in Section 3 and producing an optimized solution. It gives a schedule of conservation projects, a priority order of projects, and a schedule of fundraising.

4.1. Definitions of Expected Benefit and Cost Effectiveness

The cost effectiveness for the i^{th} project, which starts in the k^{th} year, is the ratio of the expected benefit of a project to its cost. It can be expressed as equation (1).

$$R_i(k_i) = \frac{\overline{B}_i(k_i)}{P_i} \tag{1}$$

Where $R_i(k_i)$ is the cost effectiveness, P_i is the total cost of the project, and $\overline{B}_i(k_i)$ is the expected benefit, which is given by equation (2).

$$\overline{B}_i = S_i(k_i) \cdot B_i + \left(1 - S_i(k_i)\right) \cdot 0 \tag{2}$$

Where $S_i(k_i)$ is the feasibility of success and B is the benefit if the project succeeds. (The probability of failure of the project is $\left(1 - S_i(k_i)\right)$, and in this case the benefit is 0.) Thus, we can get equation (3).

$$R_i(k_i) = \frac{S_i(k_i) \cdot B_i}{P_i} \tag{3}$$

The feasibility of success $S_i(k_i)$ decreases as time elapses (k_i increases,) and other variables remain the same. (The relationship between feasibility and time will be examined in details in Section 5.) Therefore, the cost effectiveness of a project decreases as the project is postponed.

4.2. Optimization Objective

We define the integrated cost effectiveness $R_{integrated}$ as the sum of the cost effectiveness of each project in (4).

$$R_{integrated} = \sum_{i=1}^{N} R_i(k_i) \tag{4}$$

Where R_i is the cost effectiveness of the i^{th} project. For the i^{th} project, suppose it starts in the k_i^{th} year, so it can be expressed as (5).

$$R_i(k_i) = \frac{B_i \cdot S_i(k_i)}{P_i} \tag{5}$$

The optimization model aims at maximizing $R_{integrated}$, that is:

$$max \sum_{i=1}^{N} \frac{B_i \cdot S_i(k_i)}{P_i} \tag{6}$$

4.3. Restraints

The maximum capacity of fund raising restricts the annual budget is shown in (7).

$$\sum_{i=1}^{N} f_i(j - k_i + 1) \le u_j \tag{7}$$

Where j is the j^{th} year, k_i is the year in which the i^{th} project starts, and $f_i(n)$ is the Funding required by the i^{th} project in its n^{th} year, $n = j - k_i + 1, n \in N^*$. If $j - k_i + 1 \le 0$ (the project has not started yet), $f_i(j - k_i + 1) = 0$, i.e,

$$\sum_{t=1}^{N} f_i\left(\frac{j - k_i + 1 + |j - k_i + 1|}{2}\right) \le u_j \tag{8}$$

Also, the sum of the cost of a project in each year is the total cost of the project, as shown in equation (9).

$$\sum_{j=1}^{T_i} f_i(j - k_i + 1) = P_i, \qquad i = 1, 2, \dots, N \tag{9}$$

4.4. Complete optimization model for determining schedule

$$\text{Objective: } \max \sum_{t=1}^{N} \frac{B_i \cdot S_i(k_i)}{P_i} \tag{10}$$

$$s.t. \begin{cases} \sum_{t=1}^{N} f_i\left(\frac{j - k_i + 1 + |j - k_i + 1|}{2}\right) \le u_j \\ \sum_{j=1}^{T_i} f_i(j - k_i + 1) = P_i \end{cases} \tag{11}$$

5. Feasibility of Success Model

The feasibility of successful conservation will decrease as time elapses because the population of the imperiled species shrinks. Ideally, all projects should not be postponed. However, as there is a budget limit, such approach is unrealistic. As some projects must be delayed, determining their feasibility of success in the k^{th} year is essential for evaluating an arrangement of projects.

5.1. Feasibility of Success as Function of Population Size for a species

The feasibility of successful conservation of a species is positively related to the size of its population, which can be expressed as (12).

$$S = \alpha ln(N) + b \ (0 \le S \le 1) \tag{12}$$

Where N is the size of population and S is the feasibility of survival. When the population size is small, an increase in it would result in a substantial increase in feasibility of survival; when the population size is large, S increases slowly to 1.

We found an example in reality to validate (12). For a particular species, *Scorzonera hispanica* L. *(Asteraceae),* the data derived from simulation are recorded in Table 2. [4]

Table 2 Survival Probability in Relation to Population Size for *Scorzonera hispanica* L. *(Asteraceae)*

Population Size	Survival Probability
3	0.263
5	0.608
7	0.612
10	0.993
12	0.999

However, as shown in Table 2, the MVP (minimum number of individuals that was sufficient to sustain 99% probability of population persistence in 100 years) of this species is 12, a value vastly below the MVPs of most species. In biology, the "50/500" rule states that a minimum population size of 50 was necessary to combat inbreeding and a minimum of 500 individuals was needed to reduce genetic drift. [5] The MVP for most species (500) is 42 times the MVP for the species, *Scorzonera hispanica* L. *(Asteraceae),* in Münzbergová's study. Thus, we proportionally increased the population sizes (See Table 3.)

Table 3 Survival Probability in Relation to Population Size for Most Species

Population Size	Survival Probability
125	0.263
208	0.608
292	0.612
417	0.993
500	0.999

The data in Table 3 are fitted with (12) in Fig. 4. The coefficients are $\begin{cases}\alpha = 0.5582 \\ b = -2.4252\end{cases}$.

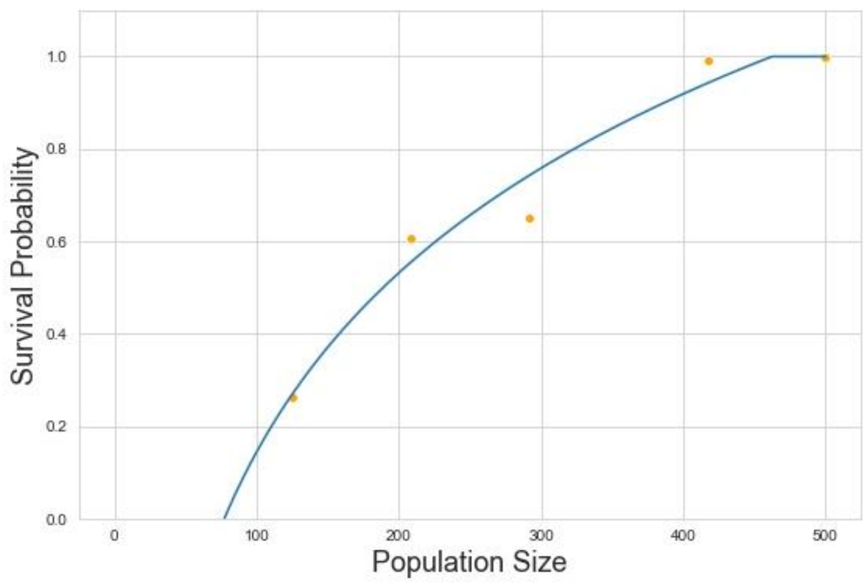

Fig. 4 Survival Probability in Relation to Population Size

5.2. Population Size as Function of Time

Suppose the birth rate and the death rate of the population is constant. The difference between them is the percentage rate of growth, r.

Typically, for an endangered species, they will extinct if no conservation actions are taken. Therefore, the percentage population growth rate r for an endangered species is negative. Since r is a constant, the rate of growth of a population is proportional to the size of the population N.

$$\frac{dN}{dt} = rN \tag{13}$$

Solving (13) results in

$$N = N_0 e^{rt} \tag{14}$$

Where N_0 is the size of the population in the first year.

We failed to find an existing and satisfactory model that predicts the growth rate r. Therefore, examples from reality were needed. A study showed that the population of *Disanthus ceercidifolius* var. *longipipes*, an endangered plant, in an area in China decreased from 2271 to 1882 in 10 years. Thus, the annual decrease is 2%. [6]

$$r = -0.02 \tag{15}$$

5.3. Population Size as Function of Time Delay

Substitute (14) into (12),

$$S = \alpha r(k-1) + b + \alpha ln(N_0) \tag{16}$$

Thus,

$$S(t) = S(0) + \alpha \cdot r \cdot t = S(0) - 0.0112t \tag{17}$$

Where $S(1)$ is the initial feasibility of success given in the dataset. In the j^{th} year, the time elapsed is $(j-1)$ years. Therefore,

$$S(j) = S(1) - 0.0112(j-1) \tag{18}$$

For the i^{th} project that starts in the k^{th} year,

$$j = k_i \tag{19}$$

6. Solving the Optimization Model

6.1. Model Solving Algorithm based on Monte Carlo Method

We set a priority order of the conservation projects instead of directly arranging the conservation projects into the years.

Each project starts as soon as the following two conditions are met: firstly, the projects prior to it are all started; secondly, starting the project would not exceed the budget limit. Given the two conditions, when the order of the conservation projects is determined, and the budget limit of each year is also determined, each priority order can only result in one arrangement of projects.

However, which priority order and its corresponding arrangement is the optimum is implicit. We have to enumerate all permutations (priority orders), obtain their corresponding arrangements, and compare the integrated cost efficiency. The arrangement with the greatest integrated cost efficiency is the optimum.

With the algorithm described above, theoretically, there are $48!$ permutations, which is roughly 1.2×10^{61}. Our computer could only calculate 1.5×10^3 cases per second, which means it was impossible to enumerate all cases. Therefore, we chose to generate permutations randomly (by Monte Carlo Method.) The model would still reach a satisfactory solution (see 6.3).

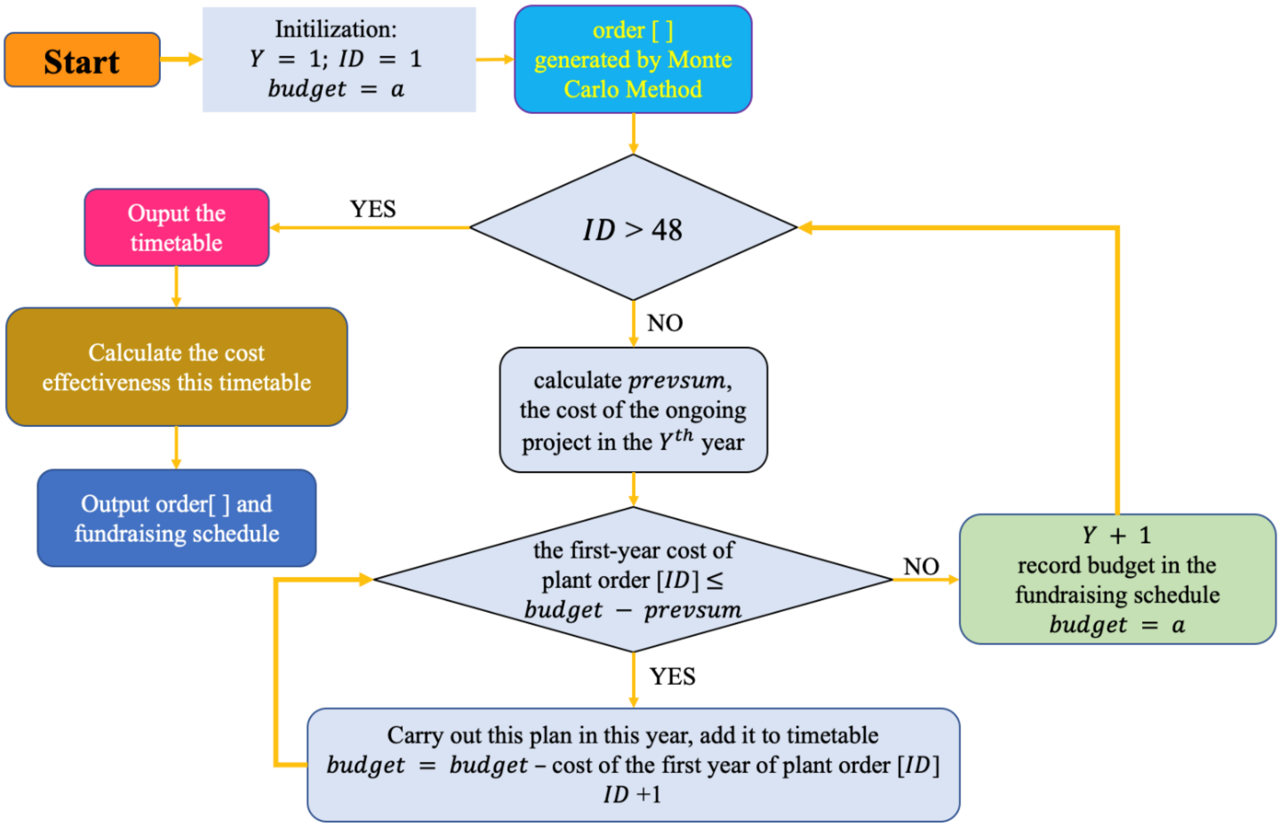

Fig. 5 Flowchart of Model Solving Algorithm through Monte Carlo Method

6.2. Demo of Algorithm

To show our algorithm more clearly, here is a small-scale demonstration in which 4 instead of 48 plants are involved, and the budgets, costs and time span of projects are simplified.

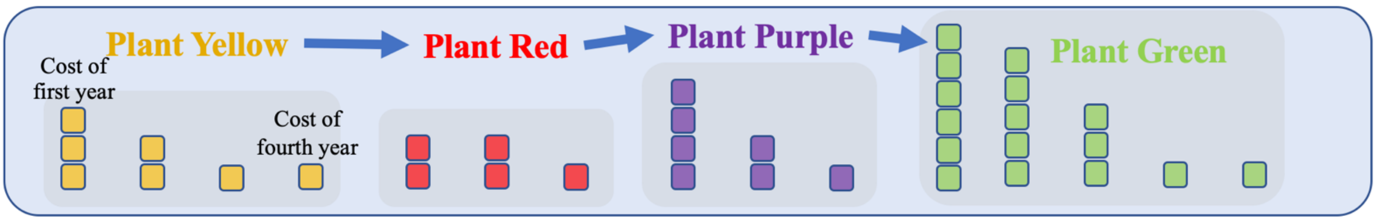

Fig. 6 Order of Protection Generated Randomly

In figure 6, the color blocks are the protection costs for various plants every year. The absolute order of protection mentioned is assigned as yellow-red-purple-green above—yellow must be protected first, and green must come last. Note that this order is only one of the $4! = 24$ possible permutations to carry the projects out.

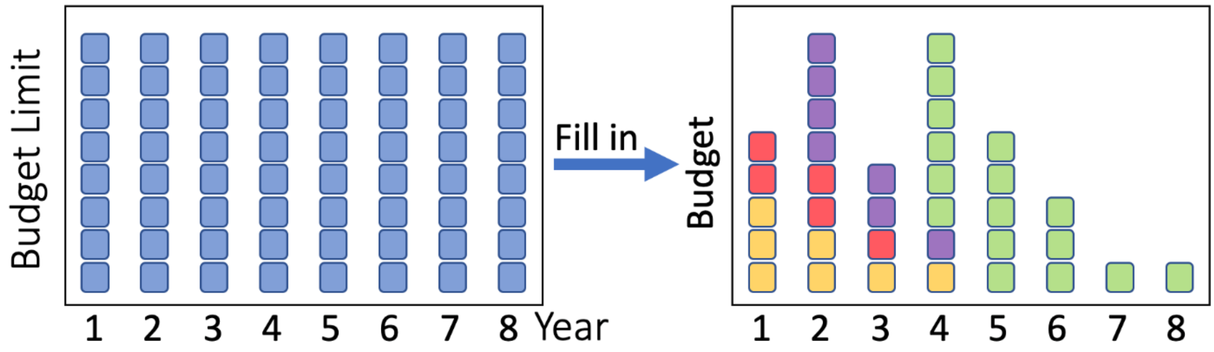

Fig. 7 & Fig. 8 Input and Output of the Demo

The conservation projects of plant yellow and red start in year 1; The purple project cannot be fitted into the budget limit of year 1. So, extra budget in year 1 is not needed (passing extra budget to the succeeding year is prohibited.)

Since plant yellow needs funding in years 1-4, the corresponding money (blocks) is marked yellow in these years. The initiation of further conservation projects must take into account of the cost of ongoing projects.

The final arrangement is thus given in Fig. 8.

This arrangement can then be rated according to the cost, feasibility and benefits of each plant. After examining the 24 permutations, the model outputs the arrangement with highest $R_{integrated}$.

6.3. Model Solution

The cost effectiveness of the current best solution is plotted in relation to the number of permutations generated in Fig. 9.

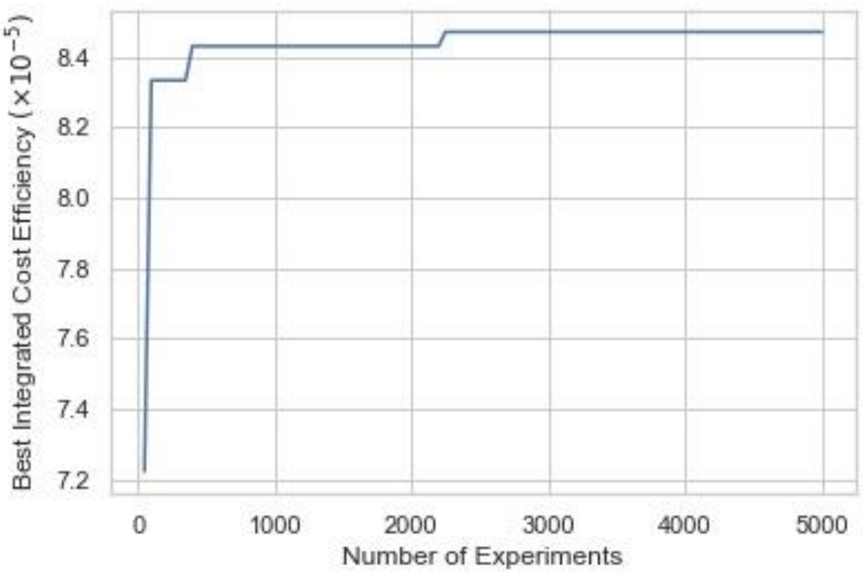

Fig. 9 Approaching the Optimum Solution with Monte Carlo Method

In the graph, the data is plotted once for each 50 experiments. Before 2200 experiments are taken, the best integrated cost effectiveness continually rises, indicating that better solutions continually emerge. Afterwards, the best integrated cost efficiency is constant—no better solution is found because the current best solution is very close to the optimum solution. The solution is then outputted.

6.3.1. Model Solution Under Constant Budget Limit

According to the financial report on the official website of FRPCE, it receives a total amount of $1,316,040 of philanthropic giving in 2019. [3] The funding, however, cannot all be put into the conservation projects of these 48 plants because other species also need protection. Suppose no more than half of the money is put into the conservation projects. Thus, the budget limit is $1,052,800.

The project with ID "Flowering Plant-415" is so expensive that its cost in the first year is two times the budget limit in reality. It is next to impossible to carry out this project, unless FRPCE raises fund for it separately. Therefore, we excluded it while arranging the rest 47 projects.

We solved the model and produced a timetable of projects, as shown in Fig. 10. The heatmap shows the cost of each project. The 48 projects are listed on the y-axis (from the least expensive to the most expensive,) and the year is marked on the x-axis. Darker color represents more expenditure.

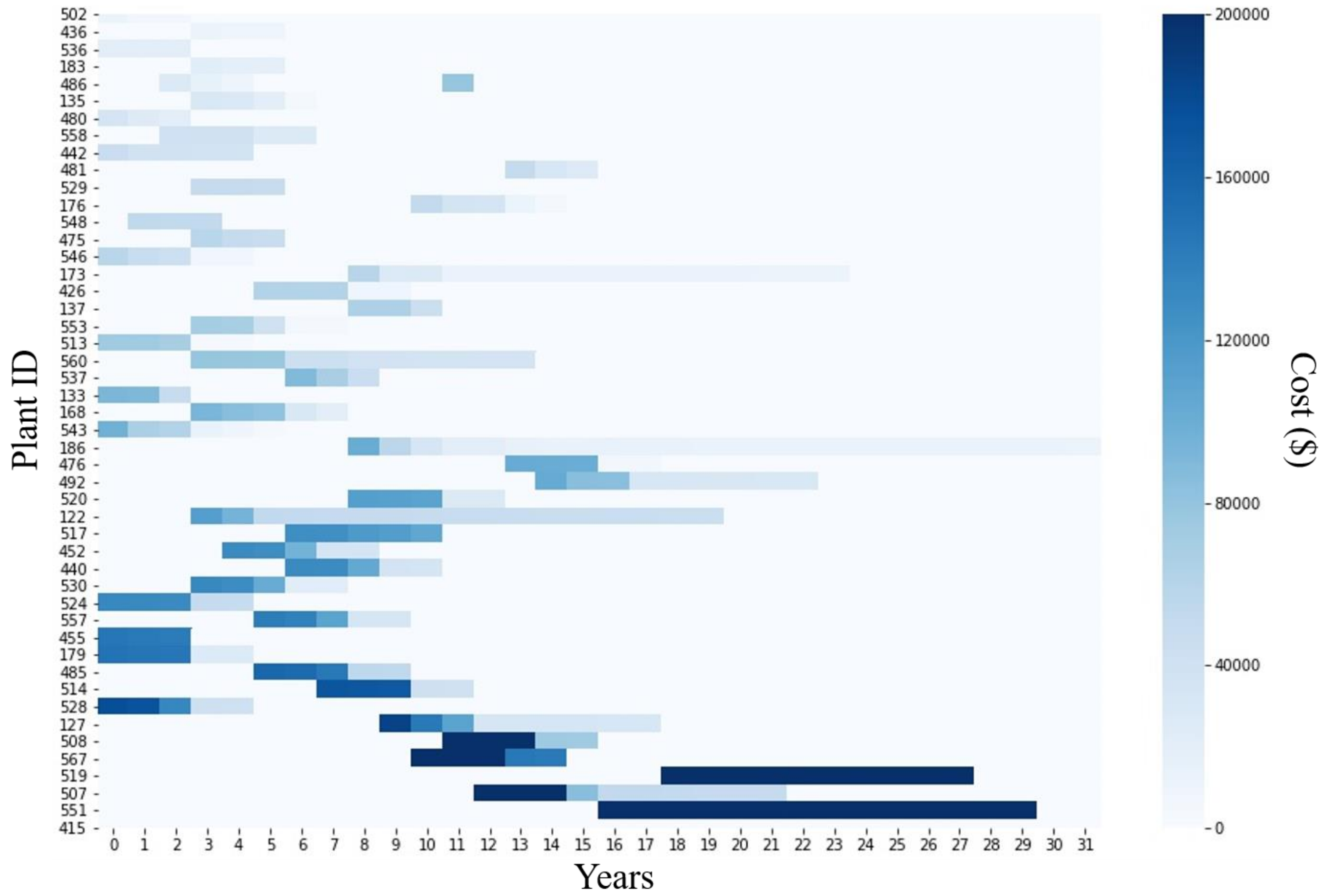

Fig. 10 Timetable of Projects and Costs Under Current Budget Limit

Because of the property of integrated cost efficiency, $R_{integrated}$, it not only maximized the cost efficiency but also prioritized the less costly projects. The cost was distributed evenly and below the maximum budget each year, as Fig. 11 shows. The yellow asterisks mark the maximum budget line, while the blue line is the annual budget required by this arrangement of projects.

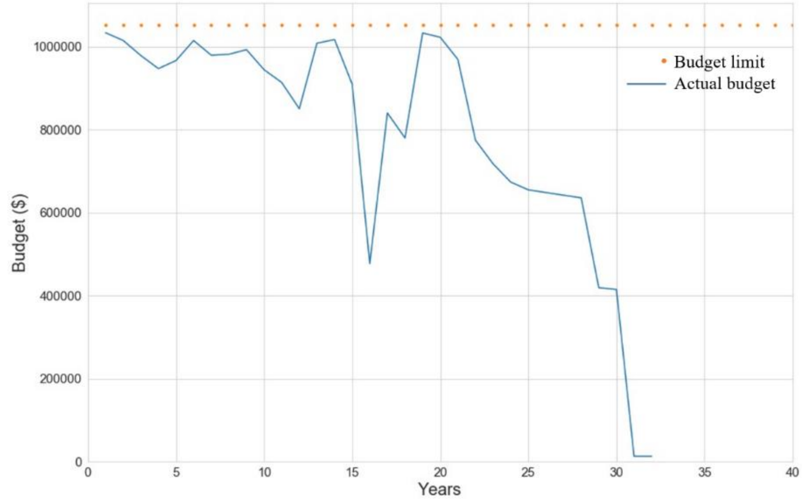

Fig. 11 Annual Budget Under Current Budget Limit

This arrangement is our proposal to FRPCE. It may arrange the conservation projects and raise funds accordingly.

6.3.2. Model Solution Under Increasing Budget Limit

If FRPCE continues to successfully protect species, it will become more influential, and fundraising will become easier each year. We generate a solution under a linearly increasing budget limit. Based on the 2018 and 2019 data, we determined the budget limit as the following:

$$u_j = 1052800 + 100800(j-1) \qquad Eq.20$$

Where j is the j^{th} year, and u_j is the budget limit of the j^{th} year.

With more budget, the projects can start earlier. Another benefit is that the previously impractical conservation project of "Flowering Plant-415" is now feasible. It starts in the 10^{th} year in our optimized solution.

The annual budgets from the Year 1 to Year 30 are shown in Fig. 12. The orange asterisks represent the budget limit, while the blue line represents the budget required.

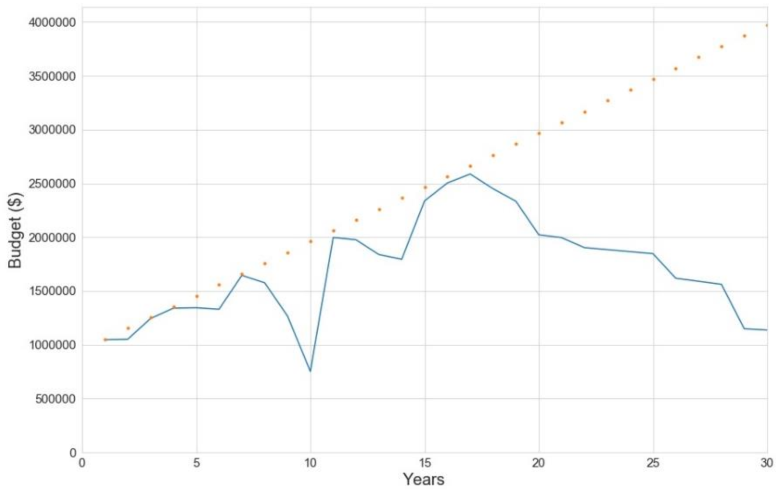

Fig. 12 Annual Budget Under Increasing Budget Limit

And the timetable of the projects and their cost is shown in the heatmap in Fig. 13.

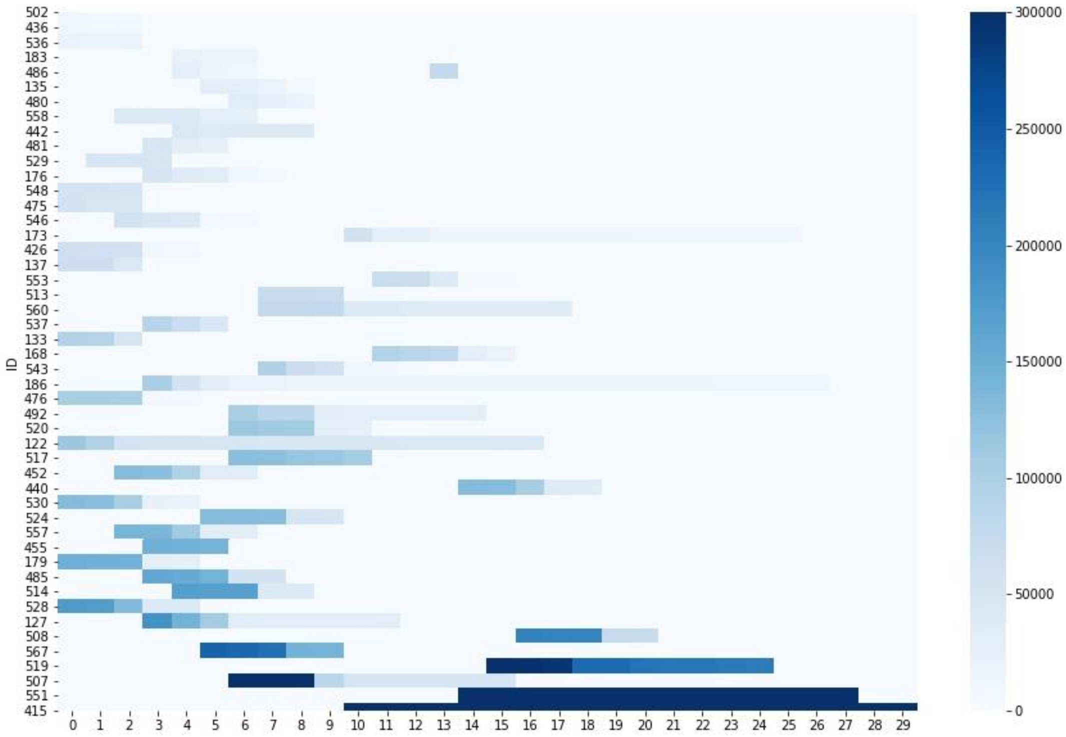

Fig. 13 Timetable of Projects and Costs Under Increasing Budget Limit

7. Sensitivity Analysis

7.1. Sensitivity Analysis of Population Growth Rate

The population growth rate r refers to the annual percentage change in size of population of an imperiled species. The value currently inputted into the model is -0.02. When r becomes more negative, $R_{integrated}$ decreases because the feasibility of success of the projects decreases faster. Also, the faster the decreasing rate is, the more the projects with higher benefit will be prioritized, as loss of postponing them becomes higher. In Fig. 14, the integrated cost efficiency is plotted in relation to r.

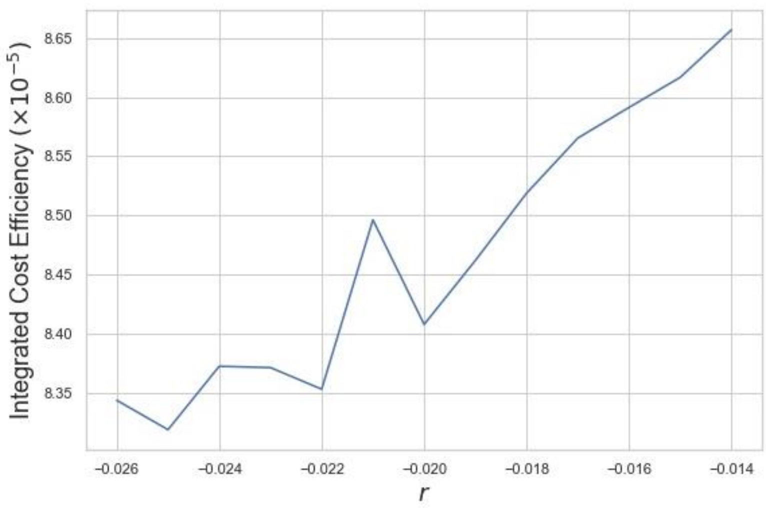

Fig. 14 Integrated Cost Effectiveness as Function of Population Growth Rate (Constant Budget Limit)

$R_{integrated}$ increases as r becomes less negative. The line fluctuates because the Monte Carlo Method includes random factors. Generally, our model can reasonably reflect the influence of r while preserving stability.

7.2. Sensitivity Analysis of Constant Budget Limit

7.2.1. Sensitivity Analysis of Budget Limit

The integrated cost effectiveness is related to the budget limit because if the budget becomes higher, the projects can start sooner. As illustrated before, the sooner a project starts, the more likely it is to succeed, and the more cost effective it is. Therefore, we expected a positive relationship between the integrated cost effectiveness and the annual budget limit. We tested our model with different budget limit, ranging from 0.658 to 3.948 million dollars (0.5 to 3 times the current annual income of FRPCE.) Subsequently, the integrated cost effectiveness rose from 7.96×10^{-5} to 9.07×10^{-5}. The model could reflect the influence of budget limit on integrated cost effectiveness reasonably.

$R_{integrated}$ is plotted in relation to annual budget limit in Fig. 15.

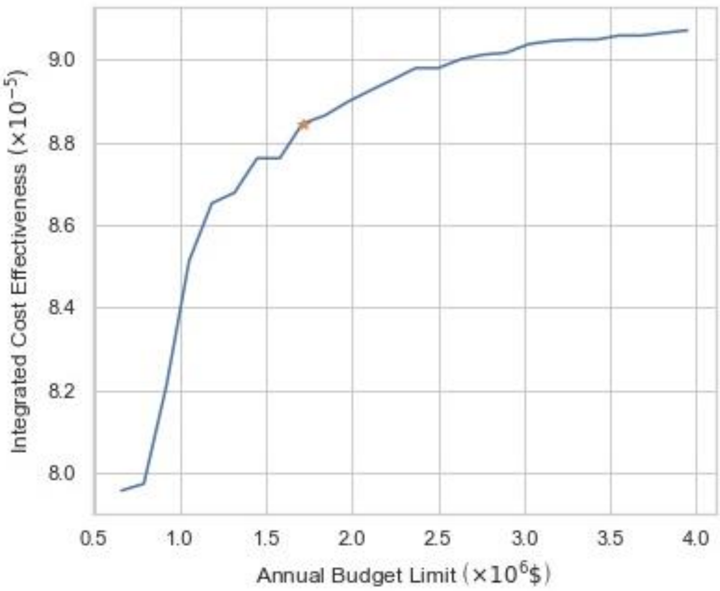

Fig. 15 Integrated Cost Effectiveness as Function of Annual Budget Limit (Constant Budget Limit)

7.2.2. Optimized Constant Budget Limit

As shown on the graph, the integrated cost efficiency rises quickly when annual budget limit is bellow 1.5 million dollars. However, the slope becomes small when the budget limit exceeds 2 million dollars. A budget of 1.71 million dollars (marked by the orange asterisk) is the most desirable. The optimum timetable when the budget limit is 1.71 million dollars, 1.3 times the current income, is shown in Fig. 16.

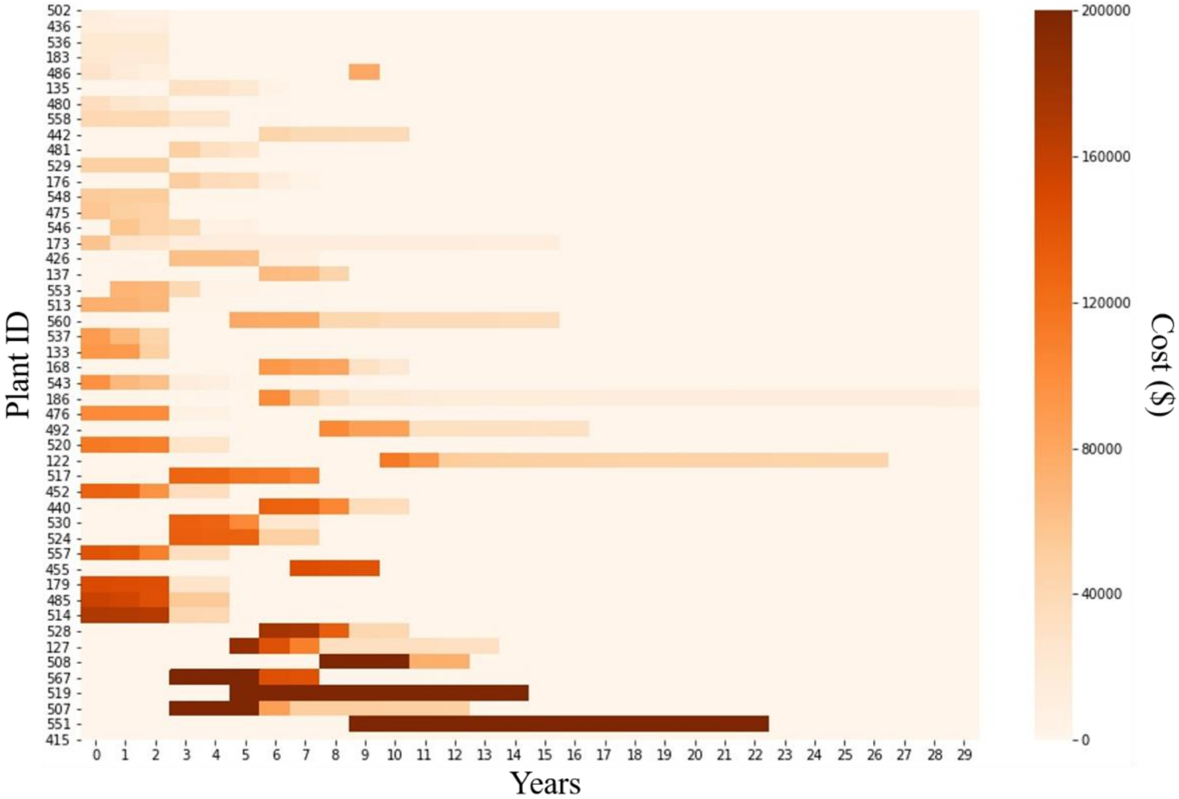

Fig. 16 Timetable of Projects and Costs Under Annual Budget Limit of $1.71M

In conclusion, the integrated cost effectiveness increases as annual budget limit increases, and the rate of increasing decreases. We chose a point where the rate of increasing vastly decreases as the ideal amount of budget. Then we presented the optimum arrangement under such budget limit.

7.3. Sensitivity Analysis of Rate of Increase of Budget Limit

Suppose the annual budget limit is linearly increasing instead of constant. The annual increase now is $100,800. When the amount of annual increase becomes larger, the integrated cost effectiveness increases because more projects can be carried out earlier. Integrated cost effectiveness is plotted in relation to the amount of annual increase in budget limit in Fig. 17.

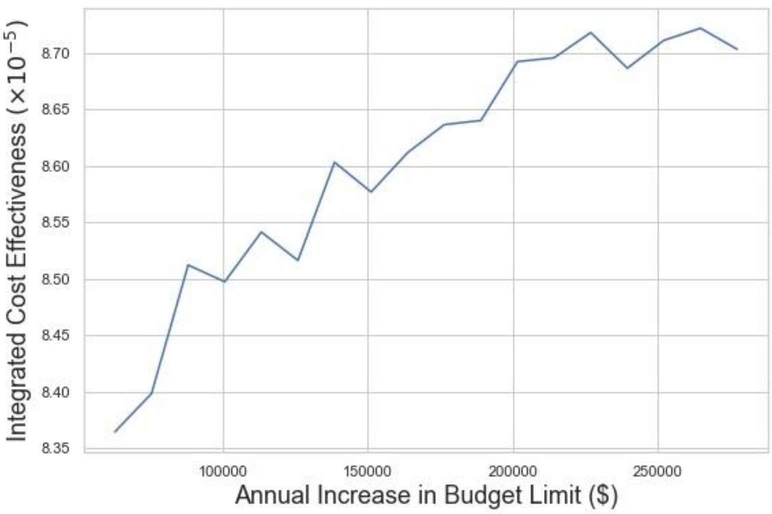

Fig. 17 Integrated Cost Effectiveness as Function of Annual Increase of Budget Limit

As the annual increase in budget limit becomes larger, the integrated cost effectiveness increases at an decreasing rate—when the annual increase in budget limit is so large that almost all projects can be started in the first few years, a further increase in the slope of budget limit will not bring as much benefit.

The fluctuation is due to the random factors in the Monte Carlo Method. Generally, our model is capable of reasonably reflecting the effect of the annual increase in budget limit while maintaining stability.

8. Strengths and Weaknesses

8.1. Strengths

- We took characters of imperiled plants into account and successfully constructed the cost effectiveness optimization model. It evaluates project arrangements objectively.
- We regarded feasibility of success as a time-dependent variable. It decreases as the conservation project is postponed. Our model is thus closer to the real-world situation.
- We solved the model by Monte Carlo method and the result was proved to be stable and satisfactory.
- We presented the solution in the form of a timetable of projects. It is straightforward and user-friendly.
- We considered project arrangements under different budget limits. We concluded that, the cost efficiency would be much higher if FRPCE can 1.3 times the money it is currently raising annually.

8.2. Weaknesses

- We had to use Monte Carlo Method while solving the model because our computers were uncapable of enumerating all possible solutions. The algorithm could potentially be improved, for example, pre-arranging the projects before running the model, in order to reach a better solution with the same amount of calculation.
- We set the budget limit as an inelastic value—the budget cannot exceed it by one cent. However, in reality, the FRPCE Board can persuade the donors to add a little more funding when there is a little shortage, or even to fund a specific project. We could build a model to take the elasticity of budget limit into account.
- We set the population growth rate r as a constant. However, in reality, the rate at which the size of population decreases is not the same among different species. It depends on the specific characteristics of each plant and the condition of its habitats. If more data were provided, we would be able to customize a value for each species.
- We did not consider redoing the conservation project of a species if the first attempt fails. In reality, the plant does not necessarily die out after the conservation project fails. The project can be carried out for the second time. We could take that possibility into account.

9. Conclusion

After noticing the trend of the data, we first established our goal of the model, that is to maximize the cost effectiveness. In order to describe the cost efficiency more accurately, we then considered the change of feasibility. A linear decline is assigned to simulate the decreasing plant population when no action is taken in a time period, which corresponds to the put off in the priority order of protection.

When solving the model, a sequence (order) of plants is first generated by Monte Carlo Method. We then assign the plants with beginning years according to the order. The year number is as small as possible, as long as the budget that year supports such arrangement. If the budget is not enough it is assigned to the next year or even later. The model then assesses the cost effectiveness of the determined schedule (arrangement). It records the best arrangement it has found. After a fixed number of rounds, we have proved that the best arrangement found will approach to a limit. The model will then output the arrangements and gives instructions on how much budget is needed every year.

We also provided the FRPCE with arrangements under different budget limits—if they want to spend only part of the budget on protecting the plants, a satisfactory solution under a lower budget limit is also available.

We determined the marginal benefit of spending more budget on these projects, which are instructive to FRPCE Board when the budget is quite limited. It helps them to get an idea of how much budget is the worthiest.

References

[1] https://time.com/5651168/trump-endangered-species-act/
[2] Negron-Ortiz, V. (2019, July 27-31). A Conservation Endowment for imperiled plants in Florida [Conference session]. Botany 2019 Conference: Sky Islands and Desert Seas, Tucson, Arizona. The Botanical Society of America.
[3] "2019 Annual Report." 2020. Fish & Wildlife Foundation of Florida. http://wildlifeflorida.org/wp-content/uploads/2020/03/25674-2019-Annual-Report_8.5x8.5_FINAL_LOW-RES.pdf. Accessed Nov. 15, 2020.
[4] Münzbergová, Z. Effect of population size on the prospect of species survival. Folia Geobot 41, 137–150 (2006). https://doi.org/10.1007/BF02806475
[5] "Minimum Viable Population." Scott K. Robinson and Carrie L. Vath. 2015. Encyclopædia Britannica. https://www.britannica.com/science/minimum-viable-population#ref1215147. Accessed Nov. 09, 2020.
[6] "STUDY ON NUMERIC DYNAMICS OF NATURAL POPULATIONS OF THE ENDANGERED SPECIES DISANTHUS CERCIDIFOLIUS VAR.LONGIPES."Yi'an Xiao et al. 2004. ACTA PHYTOECOLOGICA SINICA.
http://www.wanfangdata.com.cn/details/detail.do?_type=perio&id=zwstxb200402017. Accessed Nov. 09, 2020.

Appendix: Python Code for Solving the Model

```
slope_now = 126000
intercept_now = 1316000
time_start = time.time()
xval = [slope_now*(50+i*10)/100 for i in range(18)]
ra = [0 for i in range(18)]
coe = 20
for coeff in range(20,200,10):
    ratehis = 0
    slope = slope_now * coeff/100
    intercept = intercept_now * 80/100
    for z in range(5000):
        list1 = [i for i in range(48)]
        list1.remove(22)
        list1.remove(37)
        listx = [22,37]
        year = 0
        arrange = [[]]
        budget_ann = []
        prevsum = 0
        budget = intercept
        rate = 0
        for i in range(46):
            chosen = list1[random.randint(0,46-i-1)]
            list1.remove(chosen)
            listx.append(chosen)
        for j in range(48):
            while sheet1.cell(listx[j],6).value>budget:
                budget_ann.append(intercept+slope*year-budget)
                year = year + 1
                arrange.append([])
                prevsum = 0
                budget = intercept+slope*year
                for i in range(len(arrange)):
                    if arrange[i]!=[]:
                        for k in arrange[i]:
                            prevsum = prevsum + sheet1.cell(k,year-i+6).value
                budget = budget - prevsum
            arrange[year].append(listx[j])
            budget = budget - sheet1.cell(listx[j],6).value
        for years in arrange:
            if years != []:
                for plants in years:
                    if sheet1.cell(plants,4).value + (
```

```
arrange.index(years)*0.5582*(coe/1000))>0:
                        rate = rate + sheet1.cell(plants,2).value * (sheet1.cell(plants,4).value + (-
arrange.index(years)*0.5582*(coe/1000)))/sheet1.cell(plants,5).value*100000
        if rate>ratehis:
            ratehis = rate
    ra[int((coeff-20)/10)]=ratehis
plt.plot(xval,ra)
plt.show()
```